Steady Flow in Confined Aquifer

If there is steady movement of groundwater in confined aquifer, there will be a linear gradient/slope to the potentiometric surface, whose two directional projection is a straight line.

The quantity of flow per unit width
\[ q = Q = K \cdot b \cdot \frac{dh}{dl} \cdot a \]

The hydraulic head at any intermediate distance (a) between \( h_1 \) & \( h_2 \)

\[ h = h_1 - \frac{q'}{Kb} \cdot a \]

A confined aquifer is 33m thick and 7kms wide. Two observation wells are located 1.2 kms apart in the direction of flow. Head in well 1 is 97.5m & well 2 is 89m. K is 1.2m/day what is the daily flow of water through the aquifer? What is the elevation of potentiometric surface at a point located 300 m away from borewell 1 towards borewell 2?

Steady Flow in Unconfined Aquifer

- In the case of unconfined aquifer, the W.T. is the upper boundary of flow. It complicates the flow
- The saturated thickness in the right is \( h_1 - h_2 \)m less.
Means that the cross sectional area on the right hand side is less, hence the Head must be more to keep the steady state.

Thus the gradient of W.T in unconfined flow is not constant. It increases in the flow direction.

This problem was solved by Dupit and his assumptions are called as Dupit flow. The assumptions are:

1. The hydraulic gradient is equal to the slope of the W.T
2. For small W.T. gradients, the stream lines are horizontal and the equipotential lines are vertical. (Explain what does it mean?)

From Darcy's law,
\[ Q = -K \frac{dh}{dl} \]
h is the saturated thickness.

At \( X = 0 \), \( h = h_1 \) and at \( X = L \), \( h = h_2 \).

\[ \int_0^L q'dx = -K \int_{h_1}^{h_2} hdh \]

\[ q'x \bigg|_0^L = -K \frac{h_2^2}{2} \bigg|_{h_1} \]

Substituting the boundary conditions for \( x \) & \( h \) we get,

\[ q'L = -K \left( \frac{h_2^2}{2} - \frac{h_1^2}{2} \right) \]

\[ q' = -\frac{1}{2} K \left( \frac{h_2^2 - h_1^2}{L} \right) \]

This is called as Dupit equation of steady flow in unconfined aquifer.

If we consider small portion of the aquifer, with hydraulic gradient \( h \) on side and slope in \( x \) direction, the flow in \( x \) direction per unit width is \( q'x \).

From Darcy's law. The flow in \( x \) direction through this prism.

\[ q'dy = -K \left( h \frac{\delta h}{\delta x} \right)_x dy \]
The discharge through the right face of the prism is \( q'_{x+dx} \)

\[ q'_{x+dx} dy = -K \left( \frac{\delta h}{\delta x} \right)_{x+dx} dy \]

As \( \left( \frac{\delta h}{h \delta x} \right) \) keeps changing from face to face,

\[ (q'_{x+dx} - q') dy = -K \frac{\delta}{\delta x} \left( \frac{\delta h}{h \delta x} \right) dx dy \]

Similarly, the rate of flow in y direction,

\[ (q'_{y+dy} - q') dx = -K \frac{\delta}{\delta y} \left( \frac{\delta h}{h \delta y} \right) dy dx \]

For a steady flow, any change in flow through the prism must be equal to gain or loss across the W.T. This could be due to recharge/E.T.

If the net loss or addition is w, then the volume will be \( w dx dy \)

If w is because of E.T,

\[ -K \frac{\delta}{\delta x} \left( \frac{\delta h}{h \delta x} \right) dx dy - K \frac{\delta}{\delta y} \left( \frac{\delta h}{h \delta y} \right) dy dx = w \ dx dy \]

If we simplify the equation for dimensions,

\[ -K \left( \frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2} \right) = w \]

If \( w = 0 \) then,

\[ \frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2} = 0 \]

If flow is one directional,

\[ \frac{d^2 (h^2)}{dx^2} = -\frac{w}{K} \]

Integration of this equation \( h \), will yield the expression,

\[ h^2 = -\frac{wx^2}{K} + c_1 x + c_2 \]

At \( x = 0, h = h_1 \),

\[ x = L, h = h_2 \]

\[ \frac{w}{K} (L - x)x + h_1^2 - \frac{(h_1^2 - h_2^2)x}{L} \]

If \( w = 0 \)

\[ h_1^2 - \frac{(h_1^2 - h_2^2)x}{L} \]
An unconfined aquifer has a hydraulic conductivity of 1.73 m/day and effective porosity $n_e = 0.27$. The aquifer has a uniform thickness of 31 m. At well 1, the depth to W.T is 21 m. At well 2, located 175 m away the W.T is 23.5 m.

1. What is the discharge per unit width ($q$)?
2. What is the velocity of flow, depth to W.T midway between these two wells?

\[
q' = -\frac{1}{2}K\left(\frac{h_2^2 - h_1^2}{L}\right)
\]

\[
Q = q; \text{ at unit width } \\
A = h_1 * \text{unitwidth }
\]

\[
v = \frac{q}{n_e A}
\]

\[
h_2^2 = h_1^2 - \frac{(h_1^2 - h_2^2)x}{L}
\]

\[0.22 \text{ m}^3/\text{day per unit width}\]

\[0.80 \text{ m/day}\]

\[8.84 \text{ m}\]

### Freshwater Saline Water Relations

In coastal areas, the saline water and fresh water come into contact.

The density of saline water ($\rho_s$) is higher than the fresh water ($\rho_f$).

At coastal locations, the fresh water is discharged near the coast and mix with the saline G.W beneath ocean floor.

The mixture between fresh & saline water may be gradational or sharp.

Accordingly, generates salinity gradient. There is ground water flow both in fresh water and saline water column.

The question is at what depth the interface between the fresh and saline water is there?

Ghyben-Herzberg Principle states that

\[
Z(x,y) = (\rho_f / \rho_s - \rho_f) * h(x,y)
\]

$Z$ = depth to interface, $h(x,y)$ is the elevation of W.T (valid in static conditions)
Instead of depth to W.T., it should rather be hydraulic head

The flow in coastal aquifer is a combination of Dupitt & Ghyben-Herzberg principle

\[
\frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2} = \frac{-2w}{K (1 + G)}
\]

where

- \( w \) is the recharge to the aquifer
- \( K \) is the hydraulic conductivity
- \( G \) is equal to \( \frac{\rho_w}{\rho_s - \rho_w} \)

If the depth to salt water wedge > the aquifer thickness,

\[
\frac{\delta^2 h^2}{\delta x^2} + \frac{\delta^2 h^2}{\delta y^2} = \frac{-w}{K (z_m + h)}
\]

Where, \( z_m \) is the thickness of the aquifer below the MSL,
- \( h \) = hydraulic head
“Well is a hydraulic structure, which when designed and constructed properly permit economical withdrawal of water from a water bearing formation”.

**Static Water Level**
It is the level at which water stands before pumping starts. Its W.T.

**Piezometric surface:**
the surface to which water will rise in confined Aquifer

**Pumping water Level:**
Water level at a given rate of pumping

**Drawdown:**
SWL – PWL (H-h)

**Residual Drawdown:** (h + Δh)

**Well Yield:** LPM/ g/day

**Specific Capacity:** m³/day/drawdown

**Area of Influence:**

While pumping, PL / WT is lowered.
Area of influence
Circle of influence
Radius of influence

As pumping proceeds, the radius of influence keep increasing, till a certain distance where balance is struck between the rate of discharge and recuperation. At this stage the cone of depression stabilizes for the given rate of pumping
Open well: They derive water from the formation nearest to surface. They have large storage area (2 – 20m dia)

Tube well: (12-30 cm dia) passing through several water bearing and non water bearing formations

Flow to a well:

Let us consider flow in a well sunk in

Confined
Homogenous
Isotropic aquifer with hydraulic conductivity ‘K’

Initially the PL is horizontal

As pumping starts @ Q m³/day, head difference is generated between the well & surrounding aquifer.

Flow of water starts from aquifer to well and reduction in PL propagates outwards.

Eventually, the PL is progressively reduced sloping towards the well.

Whether the hydraulic gradient would be constant ???

No!!!!!.

1. As the water flows, it will be flowing through successively smaller area.

   As per Darcy Law

   \[ Q = K2 \pi r_1 i_1 = K2 \pi r_2 i_2 \]

   \[ 2 \pi r_1 b, 2 \pi r_2 b = \text{area}; \]

   \[ i_1 = \text{hydraulic gradient at } r_1 \text{ and } i_2 = \text{hydraulic gradient at } r_2 \]

   \[ i_1 r_1 = i_2 r_2 \]

   So \( i_1 \) must be greater than \( i_2 \) so as to maintain \( Q \)
2. Further as the water enters the well, the flow becomes turbulent and has to negotiate with the screening.

\[ \text{Drawdown} = \text{Aquifer loss} + \text{Well loss} \]

The ratio of aquifer loss to total drawdown is a measure of Efficiency of the well.

Generally we pump the well at different rates & measure the draw down & yield at each rate.

![Diagram](image1)

The yield depression curve would be a straight line if it follows Darcy’s law. But due to extra energy dissipation it will be a curve.

**Characteristics of cone of depression:**

- **Initial stage**
- **Development of Cone**
- **Stabilization of Cone**

A steady state is reached for the given rate of pumping with a characteristic radius of influence and cone of depression.

For the given aquifer the cone of depression is a function of what???

- Pumping rate,
- Pumping duration,
- aquifer characteristics,
- slope of the W.T.,
- Recharge within cone of depression

**Lesser the T, the deeper the cone**
Equilibrium well equations

In the area of influence the discharge at any distance 'r' can be given as:

\[ Q = -2\pi rbK\frac{dh}{dr} \]

\[ \left( \frac{Q}{2\pi Kb} \right) \int_{r=0}^{r_w} \frac{dr}{r} = \int_{h=0}^{h_w} dh \]

\[ h_o - h_w = \frac{Q}{2\pi Kb} \ln \frac{r_o}{r_w} \]

\[ Q = 2\pi Kb \left( h_o - h_w \right) \ln \left( \frac{r_o}{r_w} \right) \]

For a general case the well penetrating an extensive aquifer, there is no external limit for 'r' and for any given value of 'r'

Boundary conditions:
- \( h = h_w \) at \( r = r_w \)
- \( h = h_o \) at \( r = r_o \)

What is \( Kb \) ???

\[ T = Kb = \frac{Q}{2\pi (h_2 - h_1)} \ln \frac{r_2}{r_1} \]

\[ T = \frac{Q}{2\pi (s_1 - s_2)} \ln \frac{r_2}{r_1} \]

This is called as Theim Equation

Pumping must continue at a uniform rate for sufficient time to approach steady state condition.
The observation wells should be located close enough to pumping well.
Assumptions:
- Aquifer is isotropic & Steady state is reached.
- Uniform thickness
- Infinite areal extent
- Well penetrates the entire aquifer
- Laminar flow exists.
Procedure:
1. Check for steady state
On a simple logarithmic paper drawdown –time graph of each observation well is plotted. (Drawdown-vertical-linear & time-horizontal-log). Draw the time-drawdown curve through best fit. If all best fit lines run parallel in the latter part of time, mean the flow is steady state. Then we can use Theim eqn.

2. Plot the steady state draw down of each observation well against the distance ‘r’.
Draw the best fit straight line, called distance draw down curve. Find out the slope of The curve ‘ΔS’
The T can be found from the equation:

\[ Q = \frac{2\pi T (s_1 - s_2)}{\ln (r_2 / r_1)} \]

A well in a confined aquifer is pumped at a constant rate of 1500lpm. The drawdown were measured in the piezometers after 60minutes of pumping. The results are below:

<table>
<thead>
<tr>
<th>Distance of piezometer from the centre of pumping well (m)</th>
<th>3</th>
<th>9</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawdown (m)</td>
<td>6.5</td>
<td>4.75</td>
<td>3.0</td>
</tr>
</tbody>
</table>

\[ \frac{\partial}{\partial x} (K \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (K \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (K \frac{\partial h}{\partial z}) = S \frac{\partial h}{\partial t} \]

is known as the equation of unsteady ground-water flow

\[ \frac{\partial}{\partial x} (K \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y} (K \frac{\partial h}{\partial y}) + \frac{\partial}{\partial z} (K \frac{\partial h}{\partial z}) = S \frac{\partial h}{\partial t} \]

if the porous medium is isotropic with respect to hydraulic conductivity.

If it is steady state flow what will happen ??

Though K & T can be found out from Theim steady state equation, the field conditions may be such that, considerable time is required to reach steady state flow and hence, aquifer properties will have to be found out under unsteady state-flow.

Radial Flow:
Jacob, 1940 expressed the unsteady flow into a well using radial co-ordinates as:

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \]

\[ T \text{ — transmissivity of the aquifer, m}^2/\text{s} \]
\[ S \text{ — storage coefficient, dimensionless,} \]
\[ r \text{ — radial distance of the piezometer from the centre of the pumped well,} \]
\[ t \text{ — time elapsed after pumping is started, sec.} \]
Theis (1935) obtained the solution for this equation based on the analogy between groundwater flow and heat conduction, and for boundary conditions \( h = h_0 \) before pumping, and \( h \to h_0 \) as \( r \to \infty \) as pumping begins

\[
s = h_0 - h = \frac{Q}{4\pi T} \int_0^\infty e^{-u/2} du
\]

where, \( s \) — drawdown.

And

\[
u = \frac{r^2 S}{4 T T}
\]

\[
s = \frac{Q}{(4\pi T)} W(u)
\]

where, \( W(u) \) is the well function.

This is called the Theis equation.

The non-equilibrium equation permits determination of hydraulic constants \( S \) and \( T \) by means of pumping tests of wells. The equation is widely applied in practice

1. a value of \( S \) can be determined, 2. only one observation well is required, 3. a shorter period of pumping is generally necessary, and 4. no assumption of steady state flow conditions is required.

### Table 2: Values of \( W(u) \) for values of \( u \)

<table>
<thead>
<tr>
<th>( u )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
<th>( W(u) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.0 )</td>
<td>0.9947</td>
<td>0.9980</td>
<td>1.0000</td>
<td>0.9980</td>
<td>0.9947</td>
<td>0.9925</td>
<td>0.9880</td>
<td>0.9825</td>
<td>0.9750</td>
<td>0.9660</td>
<td>0.9560</td>
</tr>
<tr>
<td>( 1.1 )</td>
<td>0.9913</td>
<td>0.9947</td>
<td>1.0000</td>
<td>0.9947</td>
<td>0.9913</td>
<td>0.9880</td>
<td>0.9825</td>
<td>0.9750</td>
<td>0.9660</td>
<td>0.9560</td>
<td>0.9450</td>
</tr>
<tr>
<td>( 1.2 )</td>
<td>0.9962</td>
<td>0.9980</td>
<td>1.0000</td>
<td>0.9980</td>
<td>0.9962</td>
<td>0.9930</td>
<td>0.9880</td>
<td>0.9825</td>
<td>0.9750</td>
<td>0.9660</td>
<td>0.9560</td>
</tr>
<tr>
<td>( 1.3 )</td>
<td>0.9991</td>
<td>0.9997</td>
<td>1.0000</td>
<td>0.9997</td>
<td>0.9991</td>
<td>0.9962</td>
<td>0.9930</td>
<td>0.9880</td>
<td>0.9825</td>
<td>0.9750</td>
<td>0.9660</td>
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<td>( 1.4 )</td>
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<td>0.9999</td>
<td>1.0000</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
<td>0.9999</td>
</tr>
</tbody>
</table>

11
## Pump test Data

<table>
<thead>
<tr>
<th>Time After Pumping Started (min)</th>
<th>$t/r^2$</th>
<th>Drawdown (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$4.46 \times 10^{-6}$</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>$7.46 \times 10^{-6}$</td>
<td>0.7</td>
</tr>
<tr>
<td>8</td>
<td>$1.15 \times 10^{-5}$</td>
<td>1.3</td>
</tr>
<tr>
<td>12</td>
<td>$1.77 \times 10^{-5}$</td>
<td>2.1</td>
</tr>
<tr>
<td>20</td>
<td>$2.95 \times 10^{-5}$</td>
<td>3.2</td>
</tr>
<tr>
<td>24</td>
<td>$3.53 \times 10^{-5}$</td>
<td>3.6</td>
</tr>
<tr>
<td>30</td>
<td>$4.42 \times 10^{-5}$</td>
<td>4.1</td>
</tr>
<tr>
<td>38</td>
<td>$5.57 \times 10^{-5}$</td>
<td>4.7</td>
</tr>
<tr>
<td>47</td>
<td>$6.94 \times 10^{-5}$</td>
<td>5.1</td>
</tr>
<tr>
<td>50</td>
<td>$7.41 \times 10^{-5}$</td>
<td>5.3</td>
</tr>
<tr>
<td>60</td>
<td>$8.85 \times 10^{-5}$</td>
<td>5.7</td>
</tr>
<tr>
<td>70</td>
<td>$1.03 \times 10^{-4}$</td>
<td>6.1</td>
</tr>
<tr>
<td>80</td>
<td>$1.18 \times 10^{-4}$</td>
<td>6.3</td>
</tr>
<tr>
<td>90</td>
<td>$1.33 \times 10^{-4}$</td>
<td>6.7</td>
</tr>
<tr>
<td>100</td>
<td>$1.47 \times 10^{-4}$</td>
<td>7.0</td>
</tr>
<tr>
<td>130</td>
<td>$1.92 \times 10^{-4}$</td>
<td>7.5</td>
</tr>
<tr>
<td>160</td>
<td>$2.36 \times 10^{-4}$</td>
<td>8.3</td>
</tr>
<tr>
<td>200</td>
<td>$2.95 \times 10^{-4}$</td>
<td>8.5</td>
</tr>
<tr>
<td>260</td>
<td>$3.83 \times 10^{-4}$</td>
<td>9.2</td>
</tr>
<tr>
<td>320</td>
<td>$4.72 \times 10^{-4}$</td>
<td>9.7</td>
</tr>
<tr>
<td>380</td>
<td>$5.62 \times 10^{-4}$</td>
<td>10.2</td>
</tr>
<tr>
<td>500</td>
<td>$7.35 \times 10^{-4}$</td>
<td>10.9</td>
</tr>
</tbody>
</table>

Theis Type curve between $1/u$ & $w(u)$

Pump test plot between $h_0 - h$ & $t/r^2$

Both plots need to be in the same scale
Match the Type curve with the pump test plots by sliding the test plots on type curves, maintaining the axes parallel.

From this, for any arbitrary point of match one can find \( W(u), u, t/r^2, h_0 - h \)

By substituting in the Theis equation, one can find \( T & S \)

---

**Cooper & Jacob Method**

For small values of ‘\( r \)’ and large values of ‘\( t \)’, higher values of infinite series in Theis equation is small and can be neglected.

\[
S = \frac{Q}{4\pi T} \left( -0.5772 - \ln \frac{r^2 S}{4Tt} \right)
\]

simplifying by changing to decimal logarithms

\[
S = \frac{2.30 Q}{4\pi T} \log_{10} \frac{2.25 Tt}{r^2 S}
\]

plot of drawdown vs time is shown on a semi log paper ‘\( t \)’ forms a straightline. Projecting this line to \( S = 0 \) where \( t = t_o \)

\[
o = \frac{2.30 Q}{4\pi} \log_{10} \frac{2.25 Tt_o}{r^2 S}
\]
From this relation 'S' can be computed.

\[
s_2 - s_1 = \frac{2.3 Q}{4\pi t} \log_{10} \left( \frac{t_2}{t_1} \right)
\]

\[t_1, t_2 \text{ are chosen at one log cycle.} \]

\[\log \left( \frac{t_2}{t_1} \right) = 1, \text{ and if } s_2 - s_1 = \Delta s, \]

\[
\Delta s = \frac{2.3Q}{4\pi t}
\]

or

\[
T = \frac{2.3Q}{4\pi \Delta s}
\]

From this relation 'S' can be computed.

A Well is located in an aquifer with a conductivity of 15m/day and a storitivity of 0.005. The aquifer is 20m thick and is pumped at a rate of 2725 m³/day. What is the drawdown at a distance of 7m from the well after one day of pumping?

\[K = 15\text{m/day}, \ b = 20\text{m}, \ r = 7\text{m} \]
\[T = 300\text{m}^2/\text{day} \]

\[u = \frac{r^2 S}{4T t} = 0.0002. \]
From the table of \( W(u) \) & \( u \), if \( u = 2 \times 10^{-4} \), \( w(u) = 7.94 \)

\[
s = \frac{Q}{(4\pi T)} W(u)
\]
The saturated thickness of a non leaky, isotropic, artesian aquifer infinite in aerial extent is 55m. A production well fully penetrating the aquifer was pumped at a constant rate of 3200lpm for a period of one day. The drawdown observed at a distance of 150m from the production well in an observation well. Compute Transmissibility (T), Storitivity (S) and hydraulic conductivity (K) using Jacob & Coopers approach.

<table>
<thead>
<tr>
<th>Time after pumping started(min.)</th>
<th>Drawdown (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.34</td>
</tr>
<tr>
<td>8</td>
<td>0.39</td>
</tr>
<tr>
<td>10</td>
<td>0.43</td>
</tr>
<tr>
<td>20</td>
<td>0.53</td>
</tr>
<tr>
<td>40</td>
<td>0.65</td>
</tr>
<tr>
<td>60</td>
<td>0.70</td>
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<tr>
<td>80</td>
<td>0.75</td>
</tr>
<tr>
<td>100</td>
<td>0.88</td>
</tr>
<tr>
<td>200</td>
<td>0.88</td>
</tr>
<tr>
<td>400</td>
<td>0.97</td>
</tr>
<tr>
<td>600</td>
<td>1.02</td>
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<tr>
<td>800</td>
<td>1.05</td>
</tr>
<tr>
<td>1000</td>
<td>1.08</td>
</tr>
<tr>
<td>1440</td>
<td>1.14</td>
</tr>
</tbody>
</table>